*Contemporary Music Review*, 2002, Vol. 21, Nos 2/3, 45–52

# Theory of Groups: In Search of a Method to Analyze Group Structures in the Music of Xenakis

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The use of abstract groups marked an important change in Xenakis' compositional processes. Whereas "stochastic music" relies on indeterminism, group structures organize predetermined elements according to a specific order. Xenakis has used these structures in many works. However, he has given very little information on how to analyze them apart from in *Nomos Alpha*.[1] Because of this, most articles on this subject confine themselves to that particular work.[2] The purpose of this paper is to present an analytical tool that permits us to identify certain processes related to the theory of groups, and consequently enables us to widen the study of group structures in Xenakis' music. A complete overview of the mathematical aspect of groups falls outside the scope of this article.[3]

KEYWORDS: Xenakis, theory of groups, *Épicycle*, *Nomos Gamma*

## I. Symmetric Group $P_4$

Among the various abstract groups which Xenakis made use of, the symmetric group $P_4$ plays an important role. Formed by all possible arrangements of four elements, it is isomorphic to the octahedral group, associated with the rotations of a cube about its symmetrical axes. In this case, eight numeral values are assigned to each transformation, corresponding to the eight vertices of the cube. Since we will restrict our study to the symmetric group $P_4$, the values 5 to 8 will be ignored. Table 1 shows the values assigned to each transformation.

**Table 1**
**Values assigned to each transformation.**

| | | | |
|---|---|---|---|
| I | 1234 5678 | $Q_1$ | 7865 3421 |
| A | 2143 6587 | $Q_2$ | 7658 3214 |
| B | 3412 7856 | $Q_3$ | 8675 4231 |
| C | 4321 8765 | $Q_4$ | 6785 2341 |
| D | 2314 6758 | $Q_5$ | 6857 2413 |
| $D^2$ | 3124 7568 | $Q_6$ | 6578 2134 |
| E | 2431 6875 | $Q_7$ | 8756 4312 |
| $E^2$ | 4132 8576 | $Q_8$ | 7586 3142 |
| G | 3241 7685 | $Q_9$ | 5876 1432 |
| $G^2$ | 4213 8657 | $Q_{10}$ | 5768 1324 |
| L | 1342 5786 | $Q_{11}$ | 8567 4123 |
| $L^2$ | 1423 5867 | $Q_{12}$ | 5687 1243 |

*Contemporary Music Review*
ISSN 0749-4467 print/ISSN 1477-2256 online © 2002 Taylor & Francis Ltd
http://www.tandf.co.uk/journals
DOI:0749446022000022620

**Table 2**
**Matrix of the symmetric group $P_4$.**

| | I | A | B | C | D | $D^2$ | E | $E^2$ | G | $G^2$ | L | $L^2$ | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ | $Q_7$ | $Q_8$ | $Q_9$ | $Q_{10}$ | $Q_{11}$ | $Q_{12}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| I | I | A | B | C | D | $D^2$ | E | $E^2$ | G | $G^2$ | L | $L^2$ | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ | $Q_7$ | $Q_8$ | $Q_9$ | $Q_{10}$ | $Q_{11}$ | $Q_{12}$ |
| A | A | I | C | B | G | L | $G^2$ | $L^2$ | D | E | $D^2$ | $E^2$ | $Q_7$ | $Q_4$ | $Q_5$ | $Q_2$ | $Q_3$ | $Q_{12}$ | $Q_1$ | $Q_{10}$ | $Q_{11}$ | $Q_8$ | $Q_9$ | $Q_6$ |
| B | B | C | I | A | $L^2$ | E | $D^2$ | G | $E^2$ | L | $G^2$ | D | $Q_6$ | $Q_9$ | $Q_8$ | $Q_{11}$ | $Q_{10}$ | $Q_1$ | $Q_{12}$ | $Q_3$ | $Q_2$ | $Q_5$ | $Q_4$ | $Q_7$ |
| C | C | B | A | I | $E^2$ | $G^2$ | L | D | $L^2$ | $D^2$ | E | G | $Q_{12}$ | $Q_{11}$ | $Q_{10}$ | $Q_9$ | $Q_8$ | $Q_7$ | $Q_6$ | $Q_5$ | $Q_4$ | $Q_3$ | $Q_2$ | $Q_1$ |
| D | D | $L^2$ | $E^2$ | G | $D^2$ | I | C | L | E | A | B | $G^2$ | $Q_3$ | $Q_6$ | $Q_4$ | $Q_1$ | $Q_{11}$ | $Q_{10}$ | $Q_8$ | $Q_9$ | $Q_7$ | $Q_2$ | $Q_{12}$ | $Q_5$ |
| $D^2$ | $D^2$ | $G^2$ | L | E | I | D | G | B | C | $L^2$ | $E^2$ | A | $Q_4$ | $Q_{10}$ | $Q_1$ | $Q_3$ | $Q_{12}$ | $Q_2$ | $Q_9$ | $Q_7$ | $Q_8$ | $Q_6$ | $Q_5$ | $Q_{11}$ |
| E | E | L | $G^2$ | $D^2$ | B | $L^2$ | $E^2$ | I | A | D | G | C | $Q_{11}$ | $Q_5$ | $Q_6$ | $Q_8$ | $Q_7$ | $Q_9$ | $Q_2$ | $Q_{12}$ | $Q_3$ | $Q_1$ | $Q_{10}$ | $Q_4$ |
| $E^2$ | $E^2$ | G | D | $L^2$ | $G^2$ | C | I | E | L | B | A | $D^2$ | $Q_{10}$ | $Q_7$ | $Q_9$ | $Q_{12}$ | $Q_2$ | $Q_3$ | $Q_5$ | $Q_4$ | $Q_6$ | $Q_{11}$ | $Q_1$ | $Q_8$ |
| G | G | $E^2$ | $L^2$ | D | L | A | B | $D^2$ | $G^2$ | I | C | E | $Q_5$ | $Q_{12}$ | $Q_2$ | $Q_7$ | $Q_9$ | $Q_8$ | $Q_{10}$ | $Q_{11}$ | $Q_1$ | $Q_4$ | $Q_6$ | $Q_3$ |
| $G^2$ | $G^2$ | $D^2$ | E | L | C | $E^2$ | $L^2$ | A | I | G | D | B | $Q_9$ | $Q_3$ | $Q_{12}$ | $Q_{10}$ | $Q_1$ | $Q_{11}$ | $Q_4$ | $Q_6$ | $Q_5$ | $Q_7$ | $Q_8$ | $Q_2$ |
| L | L | E | $D^2$ | $G^2$ | A | G | D | C | B | $E^2$ | $L^2$ | I | $Q_2$ | $Q_8$ | $Q_7$ | $Q_5$ | $Q_6$ | $Q_4$ | $Q_{11}$ | $Q_1$ | $Q_{10}$ | $Q_{12}$ | $Q_3$ | $Q_9$ |
| $L^2$ | $L^2$ | D | G | $E^2$ | E | B | A | $G^2$ | $D^2$ | C | I | L | $Q_8$ | $Q_1$ | $Q_{11}$ | $Q_6$ | $Q_4$ | $Q_5$ | $Q_3$ | $Q_2$ | $Q_{12}$ | $Q_9$ | $Q_7$ | $Q_{10}$ |
| $Q_1$ | $Q_1$ | $Q_7$ | $Q_{12}$ | $Q_6$ | $Q_9$ | $Q_5$ | $Q_8$ | $Q_2$ | $Q_{11}$ | $Q_{10}$ | $Q_3$ | $Q_4$ | A | $L^2$ | $D^2$ | $E^2$ | L | B | I | $G^2$ | G | E | D | C |
| $Q_2$ | $Q_2$ | $Q_{11}$ | $Q_9$ | $Q_4$ | $Q_{10}$ | $Q_6$ | $Q_1$ | $Q_8$ | $Q_3$ | $Q_{12}$ | $Q_7$ | $Q_5$ | E | I | G | C | $L^2$ | $D^2$ | L | $E^2$ | B | D | A | $G^2$ |
| $Q_3$ | $Q_3$ | $Q_8$ | $Q_5$ | $Q_{10}$ | $Q_7$ | $Q_{11}$ | $Q_9$ | $Q_6$ | $Q_{12}$ | $Q_2$ | $Q_4$ | $Q_1$ | $L^2$ | $G^2$ | I | L | B | $E^2$ | D | A | E | C | $D^2$ | G |
| $Q_4$ | $Q_4$ | $Q_9$ | $Q_{11}$ | $Q_2$ | $Q_8$ | $Q_{12}$ | $Q_7$ | $Q_{10}$ | $Q_5$ | $Q_6$ | $Q_1$ | $Q_3$ | $G^2$ | A | D | B | $E^2$ | L | $D^2$ | $L^2$ | C | G | I | E |
| $Q_5$ | $Q_5$ | $Q_{10}$ | $Q_3$ | $Q_8$ | $Q_1$ | $Q_9$ | $Q_{11}$ | $Q_{12}$ | $Q_6$ | $Q_4$ | $Q_2$ | $Q_7$ | $E^2$ | E | A | $D^2$ | C | $L^2$ | G | I | $G^2$ | B | L | D |
| $Q_6$ | $Q_6$ | $Q_{12}$ | $Q_7$ | $Q_1$ | $Q_2$ | $Q_{10}$ | $Q_3$ | $Q_9$ | $Q_4$ | $Q_5$ | $Q_8$ | $Q_{11}$ | C | D | E | G | $G^2$ | I | B | L | $E^2$ | $D^2$ | $L^2$ | A |
| $Q_7$ | $Q_7$ | $Q_1$ | $Q_6$ | $Q_{12}$ | $Q_{11}$ | $Q_3$ | $Q_{10}$ | $Q_4$ | $Q_9$ | $Q_8$ | $Q_5$ | $Q_2$ | I | $E^2$ | L | $L^2$ | $D^2$ | C | A | E | D | $G^2$ | G | B |
| $Q_8$ | $Q_8$ | $Q_3$ | $Q_{10}$ | $Q_5$ | $Q_{12}$ | $Q_4$ | $Q_2$ | $Q_1$ | $Q_7$ | $Q_9$ | $Q_{11}$ | $Q_6$ | D | L | B | $G^2$ | I | G | $L^2$ | C | $D^2$ | A | E | $E^2$ |
| $Q_9$ | $Q_9$ | $Q_4$ | $Q_2$ | $Q_{11}$ | $Q_5$ | $Q_1$ | $Q_6$ | $Q_3$ | $Q_8$ | $Q_7$ | $Q_{12}$ | $Q_{10}$ | $D^2$ | B | $E^2$ | A | D | E | $G^2$ | G | I | $L^2$ | C | L |
| $Q_{10}$ | $Q_{10}$ | $Q_5$ | $Q_8$ | $Q_3$ | $Q_6$ | $Q_2$ | $Q_4$ | $Q_7$ | $Q_1$ | $Q_{11}$ | $Q_9$ | $Q_{12}$ | G | $D^2$ | C | E | A | D | $E^2$ | B | L | I | $G^2$ | $L^2$ |
| $Q_{11}$ | $Q_{11}$ | $Q_2$ | $Q_4$ | $Q_9$ | $Q_3$ | $Q_7$ | $Q_{12}$ | $Q_5$ | $Q_{10}$ | $Q_1$ | $Q_6$ | $Q_8$ | L | C | $L^2$ | I | G | $G^2$ | E | D | A | $E^2$ | B | $D^2$ |
| $Q_{12}$ | $Q_{12}$ | $Q_6$ | $Q_1$ | $Q_7$ | $Q_4$ | $Q_8$ | $Q_5$ | $Q_{11}$ | $Q_2$ | $Q_3$ | $Q_{10}$ | $Q_9$ | B | G | $G^2$ | D | E | A | C | $D^2$ | $L^2$ | L | $E^2$ | I |

Once the elements of the group are defined, a matrix is formed in order to calculate the result of any two consecutive transformations (table 2). As an operational table, the matrix is to be read in the sense: column – row. For example, the result of the product **A** * **D** figures at the intersection of column **A** with row **D**: $\mathbf{L}^2$. Both the table showing the values assigned to each transformation and the matrix are essential tools for the analysis of group structures. We shall often refer to them.

## II. Kinetic Diagrams

With respect to Xenakis's terminology, the choice of a specific group and the mapping of its elements with musical parameters concern the "outside-time" level. As for the succession of transformations, the "inside-time" level, Xenakis establishes a simple rule: any two consecutive transformations are followed by the one corresponding to their product. For example, a sequence that would start with transformations **A** and **D** would be continued by the product of **A** and **D**: $\mathbf{L}^2$, then the product of **D** and $\mathbf{L}^2$: **E**, etc. This process forms cyclical paths, also described as kinetic diagrams, which reflect the internal structure of the group. The symmetric group $P_4$ comprises seventy paths of different lengths.

## III. Analysis of Group Structures

In Xenakis' music, the theory of groups serves mainly to structure sets of permutated elements, which could be pitches, durations, ways of playing, registers, etc. In relation to the symmetric group $P_4$, the structural meaning of these arrangements depends on the number associated with each element. The main difficulty is then to discover these numbers without the help of the composer. In order to do so, we will have recourse to what we call *"derivative transformation."*

The *derivative transformation* defines the relation between two consecutive sets of elements, independently of their values. For example, transformation $\mathbf{E}^2$ is the *derivative transformation* of the sequence **B**, **D**. In other words, transformation $\mathbf{E}^2$ is the one that, following **B**, will produce **D**. On the matrix of table 2, it corresponds to the line with which column **B** produces transformation **D**: $\mathbf{E}^2$.

The interesting aspect of *derivative transformations* lies in the fact that their product recreates the original path. As shown in table 3, the products of the *derivatives* $\mathbf{E}^2$, **C**, $\mathbf{L}^2$, **D**, **L** reproduce the sequence (**B**), **D**, $\mathbf{E}^2$, $\mathbf{G}^2$, **A**, **E**. If we wish to determine the structure on which sets of permutated elements are based, we need to multiply their *derivative transformations*.

**Table 3**
**Analysis of a sequence by means of derivative transformations.**

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Transformations (T) path | B | | D | | $E^2$ | | $G^2$ | | A | | E |
| *Derivatives* (ΔT) | | $E^2$ | * | C | * | $L^2$ | * | D | * | L | |
| Product of *derivatives* | = | | D | | $E^2$ | | $G^2$ | | A | | E |

Since *derivative transformations* depend on the order of the elements, whatever their value may be, they enable us to verify if sets of permutated elements are ordered according to a determined path. Let us take an example from *Nomos Gamma*.

Between mm. 131 and 153, the piccolo, flutes, bassoons, and contrabassoons are divided into two groups (table 4). Within each group, the instruments follow one another sharing amongst themselves four ways of playing mapped with four dynamic forms of intensities (table 5). The distribution of these four ways of playing and dynamics among the instruments forms a group isomorphic to the symmetric group $P_4$. We are looking for the values assigned to the elements (ways of playing/dynamic forms) so that each set corresponds to a transformation of the group and their succession to one of the paths defined by Xenakis. The beginning of this sequence orders the sets as follows: {FSQN}, {NQSF}, and {FNQS} (figure 1).

Let us find the derivative transformation between these sets, that is, the transformation by which the first will produce the second, and the one by which the second will produce the third. The second set {NQSF} repeats the elements of the first {FSQN} in the order: (4321), which, according to table 1, corresponds to transformation (**C**); the third set {FNQS} transforms the second one {NQSF} by permutation (4123), which corresponds to transformation $\mathbf{Q}_{11}$. Since the product of the derivatives **C** and $\mathbf{Q}_{11}$ equals the transformation of the second set {NQSF} when the transformations follow a determined path, we find from the matrix of table 2 that the product of transformations $\mathbf{C} * \mathbf{Q}_{11}$: $\mathbf{Q}_9$ (1432). If {NQSF} represents transformation $\mathbf{Q}_9$, then N = 1, F = 2, S = 3 and Q = 4. The first set

**Table 4**
**Groups of instruments.**

| | |
|---|---|
| First group | Second group |
| Piccolo | Bassoon 1 |
| Flute 1 | Contrabassoon 1 |
| Flute 2 | Contrabassoon 2 |
| Bassoon 2 | Contrabassoon 3 |

**Table 5**
**Sets of ways of playing and dynamic forms.**

| Ways of playing (C) | Dynamic forms (G) |
|---|---|
| *flatterzunge* (F) | *pp* < *fff* |
| *staccato* (S) | *fff* > *fff* > |
| *quilisma** (Q) | *fff* > *p* < *fff* |
| *normal arco* (N) | *fff* > *p* |

*irregular oscillations of pitch


∾ = quilisma

**Figure 1**
**Excerpt from *Nomos Gamma*, mm. 131–133.**

{SQNF} stands for transformation $\mathbf{Q_4}$; and the permutations of the ways of playing mapped with the dynamic forms between the piccolo, the two flutes and the bassoon follow the path defined by the sequence: $\mathbf{Q_4}$, $\mathbf{Q_9}$, **A**, $\mathbf{Q_{11}}$, $\mathbf{Q_2}$, **A**. Having found the numeral values of the permutated elements, we can pursue this analysis and reveal the paths followed by each group of instruments (table 6).

## IV. Permutation – Substitution

As we have demonstrated, *derivative transformations* enable us to verify if sets of permutated elements follow one of the kinetic diagrams associated with each group. However, the study of abstract groups in Xenakis' more recent works requires us to distinguish between two operations: "permutation" and "substitution". Both have the same structure and, for that reason, are often equally used. How is a substitution different from a permutation? A substitution changes the values of the elements keeping them in the same order, while a permutation changes the order of the elements without considering their values. The substitution noted (2143) means that element 1, whatever position it occupies in the first set, will be substituted by element 2; element 2 by element 1; element 3

**Table 6**
**Analysis of group structures, *Nomos Gamma*, mm. 131–153.**

| Mm. | Flutes | | Contrabassoons | |
|---|---|---|---|---|
| | P | (C x G) | | |
| 131–136 | $Q_4$ | FSQN | | |
| | $Q_9$ | NQSF | | |
| | **A** | FNQS | | |
| | $Q_{11}$ | QNFS | | |
| | $Q_2$ | SFNQ | | |
| | **A** | FNQS | | |
| | | | P | (C x G) |
| 145–153 | $Q_{11}$ | QNFS | $Q_4$ | FSQN |
| | **I** | NFSQ | $Q_2$ | SFNQ |
| | $Q_{11}$ | QNFS | C | QSFN |
| | $Q_{11}$ | QNFS | $Q_{11}$ | QNFS |
| | **B** | SQNF | $Q_9$ | NQSF |
| | $Q_4$ | FSQN | C | QSFN |

by element 4; and element 4 by element 3. On the other hand, the permutation noted (2143) indicates that the second element of the first set, whatever its value may be, will be placed into the first position; the first element into the second position; the fourth into the third; and the third into the fourth. Table 7 shows the difference between these two operations, indicating, for each of them, the product of transformations **D** (2314) and **A** (2143).

As we can see, the two operations do not produce the same result, though they are equivalent when the transformations are commutative. The product **A** (2143) * **C** (4321) equals **B**, whether we consider these transformations as substitutions or permutations. This results from the commutativity between **A** and **C**: (**A** * **C**) = (**C** * **A**) = **B**. In fact, substitution corresponds to the reverse operation of permutation. In order that the matrix of table 2 describes a group based on the substitutions of four elements, we must replace the values assigned to each transformation by those of its inverse. In other words, we exchange the values between the couples (**D**, $\mathbf{D}^2$), (**E**, $\mathbf{E}^2$), (**G**, $\mathbf{G}^2$), (**L**, $\mathbf{L}^2$), ($\mathbf{Q}_1$, $\mathbf{Q}_7$), ($\mathbf{Q}_4$, $\mathbf{Q}_{11}$), and ($\mathbf{Q}_5$, $\mathbf{Q}_8$). The rest of the transformations are symmetrical. Table 8 compares the numeral values assigned to each transformation depending on whether the law of composition is defined as a permutation or a substitution. Note that to obtain a substitution of elements without exchanging these values, we could simply read the matrix of table 2 in the reverse order, finding the result of the product **D** * **A** at the intersection of line **D** and column **A**: $\mathbf{L}^2$.

Curiously, the latest edition of *Formalized Music* (Xenakis 1992) reproduces the values assigned to the transformations of this group replacing each symbol by its inverse. Since no change is indicated in the way to read the matrix, the law of

**Table 7**
**Comparison between substitution and permutation.**

| Substitution | Permutation |
|---|---|
| (**D**) (2314) | (**D**) (2314) |
| (**A**) (2143) | (**A**) (2143) |
| ($\mathbf{L}^2$)(1423) | (**G**) (3241) |

**Table 8**
**Values assigned to each transformation.**

| Permutations (P) | | | | Substitutions (S) | | | |
|---|---|---|---|---|---|---|---|
| I | 1234 | $Q_1$ | 3421 | I | 1234 | $Q_7$ | 3421 |
| A | 2143 | $Q_2$ | 3214 | A | 2143 | $Q_2$ | 3214 |
| B | 3412 | $Q_3$ | 4231 | B | 3421 | $Q_3$ | 4231 |
| C | 4321 | $Q_4$ | 2341 | C | 4321 | $Q_{11}$ | 2341 |
| D | 2314 | Q5 | 2413 | $D^2$ | 2314 | $Q_8$ | 2413 |
| $D^2$ | 3124 | $Q_6$ | 2134 | D | 3124 | $Q_6$ | 2134 |
| E | 2431 | $Q_7$ | 4312 | $E^2$ | 2431 | $Q_1$ | 4312 |
| $E^2$ | 4132 | $Q_8$ | 3142 | E | 4132 | $Q_5$ | 3142 |
| G | 3241 | $Q_9$ | 1432 | $G^2$ | 3241 | $Q_9$ | 1432 |
| $G^2$ | 4213 | $Q_1$0 | 1324 | G | 4213 | $Q_{10}$ | 1324 |
| L | 1342 | $Q_{11}$ | 4123 | $L^2$ | 1342 | $Q_4$ | 4123 |
| $L^2$ | 1423 | $Q_{12}$ | 1243 | L | 1432 | $Q_{12}$ | 1243 |

composition becomes a substitution – in accordance with the structure of the matrix – but not matching the rotation of the cube on the preceding page nor the examples given in the analysis of *Nomos Alpha* (cf. Xenakis 1992: 220ff.). Why have the symbols been replaced? An excerpt from *Épicycle* (1989) might give us the answer (see figure 2).

As figure 2 shows, the flute plays successive sets grouping together four pitches and four durations between mm. 30 and 33. The first four sets of pitches appear as follows: (abcd), (dabc), (acbd) and (acdb) – the pitches have arbitrarily been noted (a), (b), (c) and (d). Trying to analyze this sequence by means of *derivative transformations*, we obtain the results given in table 9.

Since the product of the *derivative transformations* $\mathbf{Q_{10}}$ and $\mathbf{Q_5}$ does not equal **E** but **B**, the law of composition cannot be a *permutation* of elements, unless the sets


**Figure 2**
**Excerpt from *Épicycle*, flute, mm. 30–33.**

**Table 9**
**Analysis of permutations.**

| Pitches | (abcd) | | (dabc) | | (acbd) | | (acdb) |
|---|---|---|---|---|---|---|---|
| Δ(P) | | $Q_{11}$ (4123) | | E (2431) | | $Q_{12}$ (1243) | |
| Product Δ(P) | | | $Q_{10}$ | * | $Q_5$ | ≠ | E |

**Table 10**
**Analysis of substitutions.**

| Pitches | (abcd) | | (dabc) | | (acbd) | | (acdb) |
|---|---|---|---|---|---|---|---|
| $\Delta(P)^{-1}$ | | $Q_4$ (4123) | | $E^2$ (2431) | | $Q_{12}$ (1243) | |
| Product $(\Delta(P))^{-1}$ | $(E^2)$ | | $Q_{12}$ | • | $Q_{11}$ | = | $D^2$ |


**Figure 3**
**Values associated with pitches and durations.**

do not follow one of the kinetic diagrams associated with the symmetric group $P_4$. If, however, we replace each derivative transformation of table 9 by its inverse, as shown by table 10, their product does generate a path defined by the transformations: $\mathbf{E}^2$, $\mathbf{Q}_{12}$, $\mathbf{Q}_{11}$, $\mathbf{D}^2$, etc.

The law of composition that structures these arrangements is a substitution, and *not* a permutation of elements. If (abcd) equals $\mathbf{E}^2$ (2431), then a = 2, b = 4, c = 3, and d = 1. Figure 3 shows the value assigned to each pitch along with those assigned to each duration.

Having found the law of composition and the numeral values assigned to each element, we can now reconstitute the paths followed by both sets (table 11).

This excerpt from *Épicycle* shows that Xenakis, at some time, changed the law of composition of the symmetric group $P_4$, perhaps to be in accordance with the literature on group theory, which usually uses the term "permutation" to denote what we described as "substitution". In any case, the easiest way to change that law was to replace each symbol by its inverse, as it appears in the newer edition of *Formalized Music* (Xenakis 1992). Otherwise, Xenakis would have had to recalculate each path. The values of the group transformations seem to have been printed according to changes made by Xenakis for later works. However, given the specific context in which these values take place (the analysis of *Nomos Alpha*), the ones previously published (Xenakis 1971) should be restored.

Keeping in mind the distinction we have made between permutation and substitution, the concept of derivative transformation enables us to analyze group structures based on permutated elements. It should be mentioned that the method we have developed concerns not only the symmetric group $P_4$, but also any group for which kinetic diagrams are defined by the same rule.

**Table 11**
**Analysis of group structures, *Épicycle*, mm. 30–33.**

| Flute | | | |
|---|---|---|---|
| Pitches | | Durations | |
| $E^2$ | 2431 | $Q_{12}$ | 1243 |
| $Q_{12}$ | 1243 | $E^2$ | 2431 |
| $Q_{11}$ | 2341 | $Q_8$ | 2413 |
| $D^2$ | 2314 | $Q_1$ | 4312 |
| $Q_5$ | 3142 | G | 3241 |
| $Q_9$ | 1432 | $Q_9$ | 1432 |
| D | 3124 | $Q_7$ | 3421 |
| $Q_7$ | 3421 | D | 3124 |
| $Q_{11}$ | 2341 | $Q_8$ | 2413 |
| E | 4132 | $Q_1$ | 1243 |
| $Q_{10}$ | 1324 | $D^2$ | 2313* |
| $Q_4$ | 41 | $Q_{11}$ | 34 |
| $Q_4$ | 4123 | $E^2$ | 2431† |
| G | 4213 | $Q_7$ | 3421 |
| $Q_7$ | 3421 | G | 4213 |
| $Q_9$ | 1432 | $Q_{10}$ | 1324 |
| $G^2$ | 3241 | E | 4132 |
| $Q_5$ | 3142 | E | 4132‡ |
| $Q_4$ | 4123 | Q11 | 2343§ |

*mm. 35–36, duration of the *C#* should have been 4.
† measure 39, curiously, transformation $\mathbf{E}^2$ is inserted between $\mathbf{Q}_7$ and $\mathbf{Q}_{11}$.
‡ measure 40, transformation **E** is repeated.
§ measure 41, the duration of the *D#* has been prolonged.

## *Acknowledgement*

I thank Éditions Salabert for permission to use excerpts of *Épicycle* and *Nomos Gamma*.

## *References*

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## *Notes*

1. Xenakis also gives some details there about group structures in *Nomos Gamma*.
2. However, references to group structures in *Psappha* can be found in Flint (1993).
3. A more details presentation of group structures can be found in Xenakis (1992), Vriend (1981), and Delio (1980).